647510 **MSC2MatC202x** **Seat No :________**

M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination

March/April - 2024 (NEW COURSE)

Complex Analysis (Core)

Time: 02:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any two out of three **(10)**

1) If $z, w \in \mathbb{C}$ then prove the following:
   1) $|z + w| \leq |z| + |w|$
   2) $\big||z| - |w|\big| \leq |z - w|$
2) Define extended complex plane and give spherical representation of it.
3) Let $M = \{S \mid S \text{ is bilinear map}\}$ then prove that M forms a group under binary operation composition.

Que.1(B) Answer any one out of two **(04)**

1) Find bilinear transformation which takes $1 \to i, 0 \to -i$ and $-1 \to 0$.
2) Let $f: G \to \mathbb{C}$ be analytic function $\ni f'(z) \neq 0; \forall z \in \mathbb{C}$ then prove that f is a conformal mapping.

Que.2(A) Answer any two out of three **(10)**

1) Suppose that $\gamma: [a, b] \to \mathbb{C}$ is a piece-wise smooth and γ is of bounded variation then prove that,
$$V(\gamma) = \int_a^b |\gamma'(t)| dt.$$
2) State and prove second version of Cauchy's integral formula.
3) Let G be an open subset of $\mathbb{C}$ and γ be any rectifiable path in G with initial and end points α & β respectively. If $f: G \to \mathbb{C}$ is continuous function with primitive $F: G \to \mathbb{C}$ then,
$$\int_\gamma f = F(\beta) - F(\alpha).$$

Que.2(B) Answer any one out of two **(04)**

1) Let $n \in \mathbb{N}$, then evaluate $\int_\gamma \left(\frac{z}{z-1}\right)^n dz$, where $\gamma(t) = 1 + e^{it}; 0 \leq t \leq 2\pi$.
2) Let $f, g: [a, b] \to \mathbb{C}$ be any continuous functions and let $\gamma: [a, b] \to \mathbb{C}$ be of bounded variation then
$$\int_a^b (\alpha f + \beta g) d\gamma = \alpha \int_a^b f d\gamma + \beta \int_a^b g d\gamma$$
$\forall \alpha, \beta \in \mathbb{C}$.

Que.3(A) **Answer any two out of three** **(10)**

1) State and prove Cauchy-Gorsat theorem.
2) Let $a \in \mathbb{C}, R > 0$ & $f: B(a,R) \to \mathbb{C}$ be analytic function $\ni |f(z)| \leq M, \forall z \in B(a,R)$ for some $0 < M \in \mathbb{R}$ tehn,
$$|f^n(a)| \leq \frac{n!\,M}{R^n}, \forall n \in \mathbb{N}.$$
3) Let G be an region in $\mathbb{C}$ and let $f: G \to \mathbb{C}$ be analytic function and suppose that the set $Z(f) = \{z \in G | f(z) = 0\}$ has a limit point in G then there is $a \in G$ such that $f^n(a) = 0$ for every $n \in \mathbb{N} \cup \{0\}$.

Que.3(B) **Answer any one out of two** **(04)**

1) Prove that every bounded and entire function reduces to a constant.
2) State Maximum Modulus theorem and Minimum Modulus theorem.

Que.4(A) **Answer any two out of three** **(10)**

1) State and prove Schwarz's lemma.
2) State and prove inverse function theorem.
3) If $\gamma : [0,1] \to C$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then
$$\frac{1}{2\pi i} \int_{\gamma} \frac{dz}{z-a}$$
is an integer.

Que.4(B) **Answer any one out of two** **(04)**

1) Let G be a region and let $f: G \to \mathbb{C}$ be a non-constant analytic function then prove that f is an open map.
2) Let $f: G \subset \mathbb{C} \to \mathbb{C}$ be analytic and one-one function then prove that $f'(z) \neq 0, \forall z \in G$.

Que.5(A) **Answer any two out of three** **(10)**

1) Let a be an isolated singularity of a function f then a is removable singularity of f if and only if $\lim_{z \to a} (z-a) f(z) = 0$.
2) State and prove Casorati-Weierstrass theorem for an essential singularity of a complex valued function.
3) State and prove argument principle.

Que.5(B) **Answer any one out of two** **(04)**

1) Determine number of zeros of $z^4 + 6z + 1$ inside $|z| = \frac{3}{2}$ & $|z| = 2$.
2) Deduce fundamental theorem of algebra using Rouche's theorem.

****